



Daffodil International University  
Faculty of Science & Information Technology  
Department of Computer Science and Engineering  
Final Examination, Spring-2024

Course Code: CSE212 Course Title: Discrete Mathematics

Level: 2 Term: 1 Batch: 64

Exam Duration: 02 Hours

Marks: 40

Answer **ALL** Questions

*[The figures in the right margin indicate the full marks and corresponding course outcomes. All portions of each question must be answered sequentially.]*

1. Consider the following adjacent matrix where a, b, c, d, e and f are the vertices of an undirected graph.

|   | a | b | c | d | e | f |
|---|---|---|---|---|---|---|
| a | 1 | 1 | 3 | 0 | 0 | 0 |
| b | 1 | 0 | 2 | 2 | 0 | 9 |
| c | 3 | 2 | 0 | 1 | 0 | 3 |
| d | 0 | 2 | 1 | 0 | 9 | 1 |
| e | 0 | 0 | 0 | 9 | 0 | 1 |
| f | 0 | 9 | 3 | 1 | 1 | 0 |

a) Construct a graph from the given matrix. [2]  
 b) Prove handshaking theorem for the graph. [3]  
 c) Prove whether the following graphs G and H are isomorphic or not? [5]

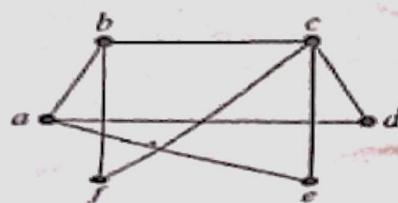

G

H

2. Derive a relation from the graph and find whether the relation is reflexive, symmetric, antisymmetric and transitive. If not make at least one relation putting the pair which are needed. [5]

b) Prove that  $2 - 2.7 + 2.7^2 - \dots + 2.(-7)^n = (1 - (-7)^{n+1})/4$  whenever  $n$  is a nonnegative integer. [5]

$$1 - \frac{(-7)^{n+1}}{4} = \frac{49}{4}$$

| 3. | <p>a) Draw the following graphs and also find out the degree of each vertices of the graphs.</p> <p>i. <math>K_{4,7}</math></p> <p>ii. <math>W_5</math></p>   | [5] | CO3 |   |    |   |   |    |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |    |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |    |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |    |   |   |    |   |   |   |  |     |
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| b) | Apply coloring algorithm to show whether the graphs are bipartite or not.   | [5] |     |   |    |   |   |    |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |    |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |    |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |    |   |   |    |   |   |   |  |     |
|    | <div style="display: flex; justify-content: space-around; align-items: center;"> <div style="text-align: center;">  <p>Graph: A</p> </div> <div style="text-align: center;">  <p>Graph: B</p> </div> </div>  |     |     |   |    |   |   |    |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |    |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |    |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |    |   |   |    |   |   |   |  |     |
| 4. | <p>Suppose that the nodes of Graph G are {a, b, c, d, e, g, h} and the weights of the edges in G are given in the following matrix. Here, the symbol (*) when there is no edge between a given pair nodes.</p> <table border="1" style="margin: auto; text-align: center;"> <tr> <th></th> <th>a</th> <th>b</th> <th>c</th> <th>d</th> <th>e</th> <th>f</th> <th>g</th> <th>h</th> </tr> <tr> <th>a</th> <td>0</td> <td>4</td> <td>5</td> <td>*</td> <td>1</td> <td>2</td> <td>1</td> <td>5</td> </tr> <tr> <th>b</th> <td>4</td> <td>0</td> <td>1</td> <td>3</td> <td>*</td> <td>1</td> <td>8</td> <td>15</td> </tr> <tr> <th>c</th> <td>5</td> <td>1</td> <td>0</td> <td>9</td> <td>2</td> <td>*</td> <td>4</td> <td>7</td> </tr> <tr> <th>d</th> <td>*</td> <td>3</td> <td>9</td> <td>0</td> <td>8</td> <td>2</td> <td>4</td> <td>6</td> </tr> <tr> <th>e</th> <td>1</td> <td>*</td> <td>2</td> <td>8</td> <td>0</td> <td>*</td> <td>*</td> <td>10</td> </tr> <tr> <th>f</th> <td>2</td> <td>1</td> <td>*</td> <td>2</td> <td>*</td> <td>0</td> <td>5</td> <td>8</td> </tr> <tr> <th>g</th> <td>1</td> <td>8</td> <td>4</td> <td>4</td> <td>*</td> <td>5</td> <td>0</td> <td>*</td> </tr> <tr> <th>h</th> <td>5</td> <td>15</td> <td>7</td> <td>6</td> <td>10</td> <td>8</td> <td>*</td> <td>0</td> </tr> </table> |     | a   | b | c  | d | e | f  | g | h | a | 0 | 4 | 5 | * | 1 | 2 | 1 | 5 | b | 4 | 0 | 1 | 3 | * | 1 | 8 | 15 | c | 5 | 1 | 0 | 9 | 2 | * | 4 | 7 | d | * | 3 | 9 | 0 | 8 | 2 | 4 | 6 | e | 1 | * | 2 | 8 | 0 | * | * | 10 | f | 2 | 1 | * | 2 | * | 0 | 5 | 8 | g | 1 | 8 | 4 | 4 | * | 5 | 0 | * | h | 5 | 15 | 7 | 6 | 10 | 8 | * | 0 |  | CO3 |
|    | a   | b   | c   | d | e  | f | g | h  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |    |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |    |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |    |   |   |    |   |   |   |  |     |
| a  | 0   | 4   | 5   | * | 1  | 2 | 1 | 5  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |    |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |    |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |    |   |   |    |   |   |   |  |     |
| b  | 4   | 0   | 1   | 3 | *  | 1 | 8 | 15 |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |    |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |    |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |    |   |   |    |   |   |   |  |     |
| c  | 5   | 1   | 0   | 9 | 2  | * | 4 | 7  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |    |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |    |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |    |   |   |    |   |   |   |  |     |
| d  | *   | 3   | 9   | 0 | 8  | 2 | 4 | 6  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |    |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |    |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |    |   |   |    |   |   |   |  |     |
| e  | 1   | *   | 2   | 8 | 0  | * | * | 10 |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |    |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |    |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |    |   |   |    |   |   |   |  |     |
| f  | 2   | 1   | *   | 2 | *  | 0 | 5 | 8  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |    |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |    |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |    |   |   |    |   |   |   |  |     |
| g  | 1   | 8   | 4   | 4 | *  | 5 | 0 | *  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |    |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |    |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |    |   |   |    |   |   |   |  |     |
| h  | 5   | 15  | 7   | 6 | 10 | 8 | * | 0  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |    |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |    |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |    |   |   |    |   |   |   |  |     |
| a) | Now, Apply Prim's algorithm, starting at node a, to construct a minimum spanning tree and find the weight based on the given matrix.  | [5] |     |   |    |   |   |    |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |    |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |    |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |    |   |   |    |   |   |   |  |     |
| b) | Apply Dijkstra on this weighted graph to find the shortest path from a to h.  | [5] |     |   |    |   |   |    |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |    |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |    |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |    |   |   |    |   |   |   |  |     |